JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

DEEP NEURAL NETWORK-BASED MODELLING OF INTERGOVERNMENTAL FISCAL TRANSFERS: AN EMPIRICAL INVESTIGATION INTO THE NATIONAL FINANCE COMMISSION AWARD AND THE CONSTITUTIONAL, FISCAL AND ADMINISTRATIVE CONTESTATIONS UNDERLYING FEDERAL DEVOLUTION IN PAKISTAN

Azam Khan[*1], Dr. Muhammad Usman Malik[2], Prof. Dr. Zulfqar Khan[3]

[*1,2,3]*National Institute of Intelligence & Security Studies, Quaid-i-Azam University, Islamabad 45320, Pakistan*

[*1]azamshaam@gmail.com, [2]usmanmalik836@gmail.com, [3]dirniiss@qau.edu.pk

DOI: https://doi.org/10.5281/zenodo.23943928

Keywords
Fiscal federalism; National Finance Commission; intergovernmental transfers; Eighteenth Amendment; deep learning; Pakistan

Article History
Received: 05 August 2026
Accepted: 19 October 2026
Published: 04 November 2026

Copyright @Author
Corresponding Author: *
Azam Khan

Abstract
Pakistan's National Finance Commission (NFC) Award is the pivot of the federal bargain, yet more than fifteen years after the Seventh Award no successor has been agreed. This article assembles an official province-year dataset of NFC transfers (FY1997–FY2027) and combines compositional data analysis, small-cluster panel econometrics and deep learning to examine how the Award has operated in practice. Transfers tracked the formula almost exactly under the 1997 Award but have departed from it persistently since 2010, with a structural break in FY2020-21; the realized allocation is consistent with a revealed population weight of about 0.68 rather than the statutory 0.82. Per-capita transfers have dispersed and then partially converged, while the 1996 Award projections overstated realized transfers by 49 log points, a pure bias error. Regularized deep neural networks and an LSTM forecast transfer growth about as well as simple benchmarks, but none is significantly better. Counterfactuals show that the 2023 census would move 3.25 points of the pool away from Punjab and that no hold-harmless redesign below one point is feasible. The findings clarify the constitutional, fiscal and administrative stakes of the Tenth NFC.

INTRODUCTION
How a federation divides its revenues is never a purely technical question. It is a continuing negotiation over political authority, identity and the terms of union (Riker 1964; Elazar 1987; Stepan 1999). In Pakistan this negotiation is institutionalized in the National Finance Commission (NFC), created by Article 160 of the 1973 Constitution, which recommends at intervals of not more than five years how the net proceeds of federal taxes are to be shared between the Federation and the provinces and among the provinces (Government of Pakistan 1973). Since the first Award of 1974, the record of concluded Awards, failed Commissions, presidential orders and year-to-year extensions has been a running

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

chronicle of the federation's unresolved tensions (Ahmad, Mustafa and Khalid 2007; Rabbani 2012; Waseem 2010).

The Seventh NFC Award, effective from 1 July 2010, and the Eighteenth Amendment of April 2010 together produced the most far-reaching redistribution of fiscal and legislative authority in the country's history (Adeney 2012; Shah 2012). The provinces' share of the net divisible pool rose from 45 per cent in FY2009-10 to 56 per cent in FY2010-11 and 57.5 per cent thereafter; the horizontal formula moved beyond population for the first time; federal collection charges fell from 5 to 1 per cent; and a new clause, Article 160(3A), prohibited any future Award from reducing the provincial share below that of the previous Award (Government of Pakistan 2010a, 2010b; Mustafa 2011). Supporters saw the settlement as the fulfilment of the federal promise to smaller provinces (Jalal 1995; Adeney 2007). Critics argued that it shifted revenue without a commensurate shift in responsibilities, narrowed the Federation's fiscal space and blunted provincial revenue effort (World Bank 2019; IMF 2019).

The Seventh Award's term expired in 2015. The Eighth and Ninth Commissions failed to agree, the Tenth was constituted in 2020, and the 2010 formula still governed the FY2026-27 budget. In the meantime, the federation has been reshaped by the merger of the former tribal areas with Khyber Pakhtunkhwa (KP) in 2018, the notification of the 2023 digital census, provincial agreement in 2024 to contribute 3 per cent of the divisible pool to federal priorities, and an FY2026-27 arrangement under which provinces return roughly Rs 1.04 trillion to the Centre as grants under Article 164. The NFC has become the focal point of three overlapping contestations: a constitutional one over the permanence of provincial entitlements, a fiscal one over vertical imbalance and budget credibility, and an administrative one over whether the formula still reflects population, need and the realities of service delivery (Cheema, Khan and Myerson 2015).

Quantitative work on Pakistan's fiscal federalism has relied largely on descriptive accounting and linear regressions (Sabir 2010; Mustafa 2011). The comparative literature on federal transfers has meanwhile moved towards designs that combine econometric identification with the flexibility of machine learning (Varian 2014; Mullainathan and Spiess 2017; Athey and Imbens 2019), and theory now establishes when deep neural networks support valid inference on economically meaningful parameters (Chernozhukov et al. 2018; Farrell, Liang and Misra 2021). The binding constraint for Pakistan is data. Province-wise transfer series are scattered across NFC reports, budget documents and economic surveys, contain gaps, and mix budgeted with realized figures. The resulting panel is small: four provinces observed over roughly two decades.

This article responds to that constraint directly. It assembles a fully sourced province-year dataset of NFC transfers covering FY1997 to FY2027, leaves gaps unimputed, flags budget estimates, and asks what can and cannot be learned from it with the most demanding tools available. The empirical strategy treats the horizontal allocation as a composition and tests formula fidelity in isometric log-ratio coordinates; it addresses cross-sectional dependence and the four-cluster problem with common correlated effects estimators and an exhaustively enumerated wild cluster bootstrap; it evaluates the credibility of ex-ante projections with Mincer–Zarnowitz tests; and it brings deep learning to bear in four ways: a softmax 'inverse-formula' network that recovers the weights implied by realized allocations, regularized deep feed-forward ensembles, a long short-term memory (LSTM) recurrent network for forecasting, and double/debiased machine learning with neural nuisance functions.

The article makes three contributions to the study of regional and federal finance. Empirically, it documents that the Seventh Award has been

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

honored in its vertical dimension but not in its horizontal dimension: realized shares deviate systematically from the notified formula, with Punjab receiving 3.7 percentage points less than its formula share on average in actual outturns, and the allocation is consistent with a population weight closer to two-thirds than to the statutory 82 per cent. Methodologically, it shows that, with data of this size, carefully regularized deep networks are competitive with but not superior to simple benchmarks, and that much of their apparent explanatory power reflects regime and reporting variables rather than economic fundamentals. Substantively, it links these findings to the constitutional, fiscal and administrative contestations surrounding the Tenth NFC and shows, through constrained optimization, that the 2023 census confronts the Commission with a redistributive trilemma that no hold-harmless formula can fully resolve.

The next section situates the NFC within theories of fiscal federalism and Pakistan's institutional history. The article then develops hypotheses, describes the data and the empirical architecture, presents the results, and discusses their implications for each of the three contestations before concluding.

Fiscal Federalism and the National Finance Commission

Theoretical Perspectives

First-generation theories of fiscal federalism assigned redistribution and stabilization to the centre and allocative functions to subnational governments, and justified transfers as instruments for closing vertical gaps, equalizing fiscal capacity and internalizing spillovers (Musgrave 1959; Oates 1972, 1999; Tiebout 1956). In this tradition, a rules-based formula is a technocratic device: it should allocate according to measurable need and capacity, be stable and predictable, and avoid weakening subnational revenue effort (Bird and Smart 2002; Boadway and Shah 2007, 2009; Bahl 2000).

Second-generation theories treat transfers as outcomes of political bargaining and as incentives that shape the behaviour of governments (Qian and Weingast 1997; Weingast 1995, 2009; Oates 2005). Transfers that are large, discretionary or renegotiable soften subnational budget constraints and invite strategic behaviour (Kornai 1986; Kornai, Maskin and Roland 2003; Rodden, Eskeland and Litvack 2003; Rodden 2006). The flypaper literature shows that grant money tends to stick where it lands rather than substituting for local taxes (Gramlich 1977; Hines and Thaler 1995; Inman 2008). Political-economy models predict that discretionary transfers flow to pivotal or aligned jurisdictions (Dixit and Londregan 1998; Khemani 2007), while comparative research stresses that the architecture of intergovernmental institutions, not only their parameters, determines whether decentralization strengthens or weakens the union (Treisman 2007; Watts 2008; Rodden 2002).

These perspectives yield different expectations for a formula-based system such as the NFC. A technocratic reading expects realized transfers to follow the notified formula and to respond, at least at each Award, to changes in measured need. A bargaining reading expects formulas to be supplemented by add-ons, straight transfers and discretionary grants that reflect political leverage, and expects the formula itself to be frozen when updating would create losers. The empirical question is which pattern describes the Pakistani case, and when.

The Evolution of the NFC

The 1973 Constitution created a divisible pool of federal taxes to be shared through the NFC, building on pre-partition arrangements (Ahmad, Mustafa and Khalid 2007; Zaidi 2015). The 1974 Award gave the provinces 80 per cent of a relatively narrow pool, distributed by population alone. The Commissions of 1979 and 1985 failed to reach agreement, and resource sharing continued on the 1974 basis during military rule (Jalal 1995; Talbot

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

2009). The 1991 Award retained the 80 per cent provincial share and population-based distribution but added special grants for every province. The 1997 Award widened the pool to all federal taxes while cutting the provincial share to 37.5 per cent, so that the two figures are not directly comparable (Figure 1). When the Sixth Commission deadlocked, President's Order No. 1 of 2006, issued under Article 160(6), raised the provincial share in steps from 41.5 to 46.25 per cent and updated population shares to the 1998 census.

The Seventh Award, signed at Gwadar in December 2009 and notified in March 2010, broke with this history in three respects. It was the first consensus Award since 1991 negotiated by elected governments; it raised the vertical share to 57.5 per cent; and it introduced a multi-criteria horizontal formula weighting population at 82 per cent, poverty and backwardness at 10.3 per cent, revenue collection and generation at 5 per cent and inverse population density at 2.7 per cent. Its side provisions mattered as much as its formula: 1 per cent of the divisible pool was reserved for KP for the war on terror, Sindh received 0.66 per cent of provincial sales tax in compensation for the abolition of the octroi and zila tax, and Balochistan was guaranteed Rs 83 billion in the first year. The Eighteenth Amendment then entrenched the vertical outcome through Article 160(3A).

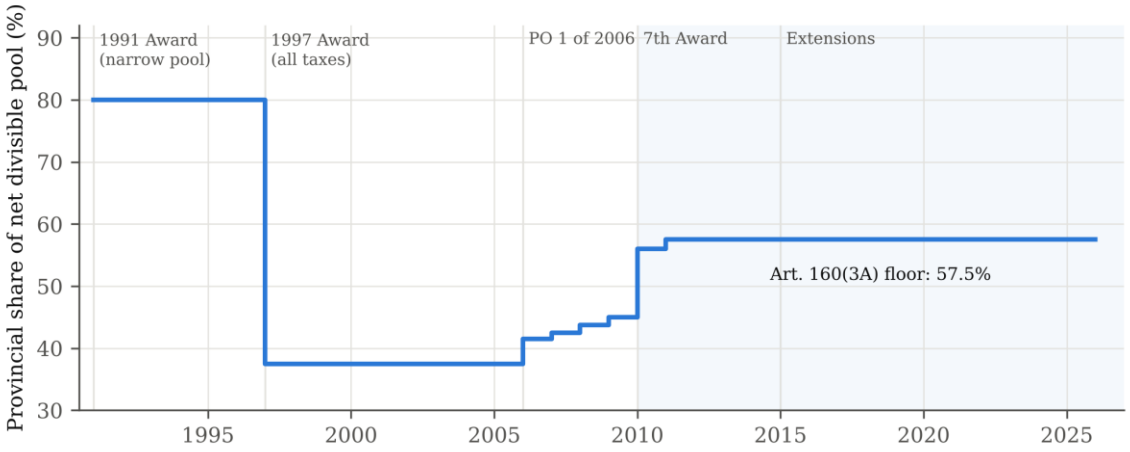


Figure 1. Provincial share of the net divisible pool under successive Awards and orders, FY1991-92 to FY2026-27. Source: Author's construction from the dataset (Vertical_Share sheet; Finance Division and NFC Report 2009). The 1991 share applies to a narrower pool and is not directly comparable with later years.

Three Contestations

The constitutional contestation concerns the status of provincial entitlements. Article 160(3A) converts the 2010 vertical share into a floor, and successive federal governments have sought to work around it by asking provinces to fund national priorities: in 2020 the Centre proposed that provinces forgo about 7 per cent of the net pool for security, the merged districts and Azad Jammu and Kashmir and Gilgit-Baltistan; in 2024 provinces agreed to contribute 3 per cent; and the FY2026-27 budget routes about Rs 1.04 trillion back to the Centre as Article 164 grants. These devices preserve the letter of the floor while reducing its substance.

The fiscal contestation concerns vertical imbalance, macroeconomic stability and the reliability of the numbers on which the bargain rests. Award projections and budget estimates are themselves instruments of negotiation; if they are

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

systematically optimistic, provinces plan against resources that do not materialize, and the credibility of the whole arrangement suffers (IMF 2019; World Bank 2019).

The administrative contestation concerns whether the formula reflects the federation as it now is. Population still enters through the 1998 census, although the 2017 and 2023 censuses record markedly different provincial shares. The merger of the former tribal areas added territory and population to KP that the 2010 formula does not cover, and the delivery of devolved functions depends on resources travelling below the provincial tier to local governments mandated by Article 140A (Cheema, Khwaja and Qadir 2006; Cheema, Khan and Myerson 2015).

Literature and Hypotheses

Studies of the Seventh Award have quantified its first-round fiscal implications (Sabir 2010; Mustafa 2011), traced its constitutional politics (Adeney 2012; Rabbani 2012; Shah 2012) and assessed its consequences for provincial revenue effort and service delivery (Bahl, Wallace and Cyan 2008; World Bank 2019). Comparative work in India, which has a structurally similar Finance Commission, shows that formula transfers are more equalizing and less politically targeted than discretionary grants (Rao and Singh 2005; Khemani 2007). Two gaps stand out. First, almost no study compares realized provincial allocations with the notified formula over time, although that comparison is the most direct test of whether the formula governs. Second, the methodological literature on machine learning in economics has rarely been applied to federal transfer systems, and where it has, the small size of the available panels has seldom been confronted directly (Mullainathan and Spiess 2017; Athey and Imbens 2019).

Five hypotheses follow from the theoretical discussion.

• **H1 (formula fidelity).** Under the 1997 Award, which relied on a single criterion and few add-ons, realized horizontal shares equal formula shares; under the Seventh Award, with its add-ons, straight transfers and extensions, they deviate systematically.

• **H2 (census rigidity).** Because the formula uses a frozen 1998 population base, realized shares do not respond to subsequent demographic change, and the divergence between applied and census-updated shares widens over time.

• **H3 (equalization).** The multi-criteria formula raised the dispersion of per-capita transfers in favour of the least populous province, after which dispersion converges as the add-ons are diluted by the growth of the pool.

• **H4 (credibility).** Ex-ante projections embedded in Awards are biased upwards, whereas annual budget estimates under the Seventh Award are unbiased predictors of outturns.

• **H5 (learnability).** Given four provinces and around two decades of data, regularized deep neural networks cannot significantly outperform simple benchmarks in forecasting transfer growth, and their attributions are dominated by regime and reporting variables.

Data and Empirical Strategy

Data

The analysis uses an original dataset compiled entirely from official documents and from press reports of those documents (Table 1). It contains the full history of NFC Awards and orders, the vertical share for every fiscal year from FY1991-92 to FY2026-27, the indicator magnitudes and weights of the Seventh Award formula, a province-year panel of federal transfers tagged as actual, budget estimate, benchmark or Award projection, federal loans and grants outside the divisible pool for FY2013-14 to FY2020-21, provincial populations from the 1981, 1998, 2017 and 2023 censuses, and a coded chronology of sixteen constitutional, fiscal and administrative contestations from 1973 to 2026. The principal sources are the Report of the National Finance Commission 2009, federal budget documents, the

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Pakistan Economic Survey and census results (Government of Pakistan, Finance Division 2009, 2024, 2025a, 2025b; Pakistan Bureau of Statistics 2023), supplemented by contemporaneous press reports of official records. Nothing is simulated or imputed. Province-wise figures for FY2002-03 to FY2008-09, FY2010-11 and FY2012-13 are not available in consistent form and are left missing.

The working panel takes, for each province and year, the actual outturn where one exists, the budget estimate otherwise, and the 7th NFC benchmark for FY2009-10. This yields 84 province-year observations in 21 fiscal years, of which 56 form a contiguous block from FY2013-14 to FY2026-27. Budget-estimate years are flagged and, where inference could be affected, analysed separately. Two features of the sources require care. Actual outturns for FY2013-14 to FY2020-21 combine divisible-pool and straight transfers, and budget figures from FY2023-24 onward do the same, while some earlier budget figures report the divisible pool only. This definitional shift is controlled for by the budget flag and is discussed where it matters. Population is interpolated log-linearly between censuses and extrapolated after 2023 at the 2017–2023 growth rate; KP's series includes the merged districts from 2017.

Table 1. Data sources, coverage and constructed variables.

Variable	Definition and construction	Coverage	Source (dataset code)
T_it	Federal transfers to province i in fiscal year t (Rs billion, nominal); actual outturn preferred, budget estimate otherwise (flagged)	FY1997–2001, 2009, 2011, 2013–2026	Finance Division; NFC Report 2009; Budget in Brief; Economic Survey (S1–S14)
s_it	Realized horizontal share, T_it / Σ_j T_jt	As above	Derived
f_it	Formula share in force under the governing Award or Order	As above	Award_History; NFC7_Formula (S1, S2, S4)
dev_it	Formula deviation, 100 × (s_it − f_it), percentage points	As above	Derived
v_t	Provinces' vertical share of the net divisible pool	FY1991–2026	Vertical_Share (S1, S2, S4, S5, S9)
pop_it	Population (millions), log-linear interpolation between the 1981, 1998, 2017 and 2023 censuses; 2017–23 CAGR thereafter	FY1997–2026	PBS censuses (S15, S16)
pc_it	Per-capita transfers, Rs per person, T_it / pop_it	As above	Derived
A_it, F_it	Actual transfer and its ex-ante counterpart (1996 Award projection; budget estimate)	FY1997–2001; FY2013, 2015, 2021	S1 (Annex VI); S10–S14
grants_it	Federal loans and grants outside the divisible pool (Rs billion)	FY2013–2020	Grants_Loans (S10)

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Variable	Definition and construction	Coverage	Source (dataset code)
Event indicators	Cumulative contestation count; 7th Award extension (2015+), FATA merger × KP (2018+), 10th NFC constituted (2020+), 2023 census (2023+), provincial contribution (2024+)	1973–2026	Contestations (S1–S5, S15–S23)

Notes: Units are nominal Rs billion unless stated. The FY2002–FY2008, FY2010 and FY2012 province-wise transfers are not available in consistent form and are not imputed. Budget-estimate years are flagged and treated separately in all inference.

Compositional Analysis of Formula Fidelity

Horizontal shares are compositions: they are positive and sum to one, so ordinary regression on raw shares is ill-posed (Aitchison 1986). Following Egozcue et al. (2003), each four-part composition s_t is mapped to three isometric log-ratio (ilr) coordinates using a Helmert basis H, and the realized–formula gap in year t is

$$z_t = \mathrm{ilr}(s_t) - \mathrm{ilr}(f_t) = H\,[\ln s_t - \ln f_t - (1/4)\Sigma_j (\ln s_{jt} - \ln f_{jt})] \quad (1)$$

Its Euclidean norm is the Aitchison distance between the realized and formula compositions. Formula fidelity (H1) implies $E(z_t) = 0$, which is tested with Hotelling's one-sample T^2 statistic. Because the samples are short, the F approximation is complemented by an exact randomization test that enumerates all 2^n sign flips of z_t. A Quandt–Andrews sup-F test (Quandt 1960; Andrews 1993) with 15 per cent trimming and bootstrap p-values tests for an unknown break in the mean Aitchison distance over FY2013-14 to FY2026-27.

Panel Econometrics with Four Clusters

Three panel questions are examined on the contiguous FY2013–2026 block. The first is whether per-capita transfers converge across provinces (H3). The β-convergence regression (Barro and Sala-i-Martin 1992) is

$$\Delta \ln pc_{it} = \alpha_i + \lambda_t + \beta \ln pc_{i,t-1} + \varepsilon_{it} \quad (2)$$

where α_i and λ_t are province and year effects. Because a lagged dependent variable with fixed effects is biased in short panels (Nickell 1981), the half-panel jackknife of Dhaene and Jochmans (2015) is reported alongside the two-way fixed-effects (TWFE) estimate. The second question is whether realized shares respond to demographic change (H2):

$$\ln s_{it} = \alpha_i + \lambda_t + \theta \ln p_{it} + u_{it} \quad (3)$$

where p_{it} is the current population share. If the formula were updated annually, θ would be close to the formula-implied elasticity 0.82 p_{it}/f_{it}, which averages 0.78; under a frozen census base θ should be zero. The third question is whether discretionary grants outside the pool respond to the FATA merger, using the KP-by-post-2018 interaction.

Inference is the central difficulty. With four provinces, cluster-robust standard errors are severely downward-biased and the usual wild cluster bootstrap has too few distinct draws under Rademacher weights (Cameron, Gelbach and Miller 2008; MacKinnon and Webb 2017). All panel tests therefore use the restricted wild cluster bootstrap with Webb's six-point weights and full enumeration of the $6^4 = 1{,}296$ possible draws (Webb 2023), together with Driscoll–Kraay standard errors, which are robust to cross-sectional dependence of unknown form (Driscoll and Kraay 1998). Cross-sectional dependence is measured with the Pesaran (2015) CD statistic, and stationarity with the CIPS test (Pesaran 2007), whose critical values are simulated for N = 4 and

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

T = 14. Heterogeneous responses are estimated by the common correlated effects mean group (CCE-MG) estimator, which proxies unobserved common factors with cross-sectional averages (Pesaran 2006; Chudik and Pesaran 2015):

$$\ln T_{it} = a_i + b_i \ln pop_{it} + c_i \ln \bar{T}_t + d_i \ln \bar{pop}_t + e_{it}, \quad \hat{b}_{MG} = N^{-1} \Sigma_i \hat{b}_i \qquad (4)$$

Budget Credibility

For H4, the realized transfer A_{it} is compared with its ex-ante counterpart F_{it}, either the projection made by the 1996 Commission for FY1997-98 to FY2001-02 or the federal budget estimate (FY2013-14, FY2015-16 and FY2021-22). The Mincer–Zarnowitz (1969) regression

$$\ln A_{it} = \alpha + \beta \ln F_{it} + u_{it}, \quad H_0: \alpha = 0, \beta = 1 \qquad (5)$$

tests forecast rationality jointly, with p-values from the enumerated wild cluster bootstrap. Theil's (1966) decomposition splits the mean squared error into bias, variance and covariance proportions, which indicate whether errors are systematic or random.

Deep Learning Architecture

Deep learning enters the analysis in four roles, each chosen for what it can credibly deliver with a small panel (Figure 2).

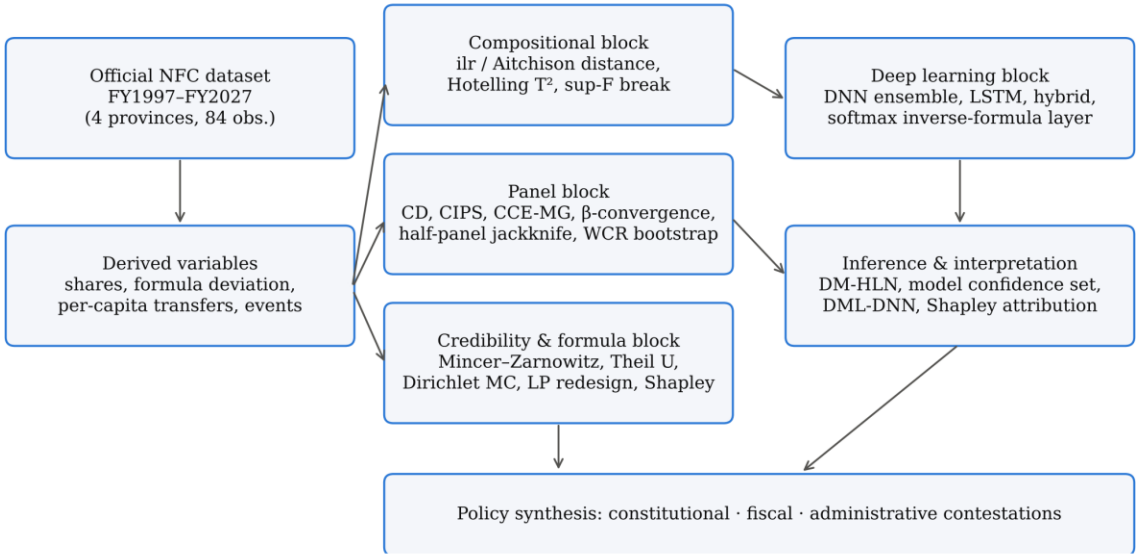


Figure 2. Integrated econometric and deep learning architecture of the study.

Source: Author.

The first is an inverse-formula network. The Seventh Award's indicator matrix M (four criteria by four provinces) is fixed, and a formula is a mapping $s = w'M$ with weights on the simplex. A single-layer network with softmax-parameterized weights,

$$w = \text{softmax}(z), \quad \hat{s} = w'M, \quad \min_z \|\hat{s} - s\|^2 \qquad (6)$$

is trained by Adam (Kingma and Ba 2015) on mean realized shares to recover the 'revealed' weights that would have produced the observed allocation. The softmax layer enforces non-negativity and adding-up by construction; a year-block bootstrap gives uncertainty bands. The exercise asks which formula the realized transfers behave as if they followed.

The second is a family of forecasting models for provincial transfer growth $g_{it} = \Delta \ln T_{it}$. Inputs are lagged own and national growth, the lagged formula deviation, lagged relative per-capita transfers, the change in population share, the

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

budget flag, event indicators for the FATA merger, the Tenth Commission, the 2023 census and the provincial contribution era, cumulative contestations and province indicators. A deep feed-forward network with ReLU activations (Goodfellow, Bengio and Courville 2016; LeCun, Bengio and Hinton 2015) is estimated as a ten-member deep ensemble (Lakshminarayanan, Pritzel and Blundell 2017). Architecture and L2 penalty are chosen at each forecast origin on the final year of the training window only, so that no information from the evaluation year leaks into model selection. A hybrid model fits the network to the residuals of a ridge regression (Zhang 2003). An LSTM network (Hochreiter and Schmidhuber 1997) with six hidden units reads two-year sequences of growth, formula deviation, relative per-capita transfers and the budget flag; it was implemented from first principles with back-propagation through time. These are benchmarked against a naive no-change forecast, the historical mean, pooled OLS, ridge regression with fixed effects, random forests (Breiman 2001) and gradient boosting. Forecasts are generated in a rolling-origin design for FY2019-20 to FY2026-27, giving 32 out-of-sample forecasts per model. Accuracy is compared with Diebold–Mariano tests (Diebold and Mariano 1995) using the Harvey, Leybourne and Newbold (1997) correction and summarized by the model confidence set (Hansen, Lunde and Nason 2011).

The third role is interpretation. Shapley values (Shapley 1953; Štrumbelj and Kononenko 2014; Lundberg and Lee 2017), estimated by interventional permutation sampling, attribute each ensemble prediction to its inputs.

The fourth is double/debiased machine learning (Chernozhukov et al. 2018). In the partially linear model $\Delta \ln pc_{it} = \theta \ln pc_{i,t-1} + g(X_{it}) + \varepsilon_{it}$, the nuisance functions $E(Y|X)$ and $E(D|X)$ are estimated by deep networks with year-block cross-fitting, and θ is obtained from the orthogonalized residuals. Deep networks satisfy the rate conditions required for valid inference under suitable smoothness (Farrell, Liang and Misra 2021); random forests and ridge regression provide comparison learners.

Formula Counterfactuals

Finally, the Seventh Award formula is re-run with 2023 census population shares, holding the other indicators at their 2010 values. Uncertainty over weights is represented by 20,000 Dirichlet draws centred on the official weights. A linear programme then searches for weights that maximize the combined weight on poverty and inverse density subject to a population weight of at least 0.70, a revenue weight of at least 0.05 and a hold-harmless constraint that no province loses more than δ percentage points relative to its current share. Each redesigned share change is decomposed into a census effect and a reweighting effect with two-factor Shapley decomposition (Shorrocks 2013), and transfers for FY2029-30 are projected by bootstrapping historical growth of the pool.

Results

Descriptive Patterns

Table 2 summarizes the data by regime. Under the 1997 Award realized shares matched the population formula to within 0.02 percentage points, and per-capita transfers were almost equal across provinces. By FY2009-10, under the 2006 Presidential Order, Punjab's realized share was already 4.35 points below its formula share. Under the Seventh Award, total transfers rose from about Rs 1 trillion in the first full budget to a mean of Rs 2.2 trillion in FY2013–2021 and Rs 6.8 trillion in the budgets of FY2022–2026, reaching Rs 8.8 trillion in FY2026-27. Over the actual-outturn years Punjab received 48.07 per cent against a formula share of 51.74 per cent, while Sindh, KP and Balochistan received more than their formula shares. Balochistan's per-capita transfers were 1.84 times Punjab's.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Table 2. Transfers, Realized Shares and Formula Deviations by Regime.

Period	Years	Mean total (Rs bn)	Punjab	Sindh	KP	Balochistan	Per-capita ratio Bal./Pun.
1997–2001 (5th Award, actual)	5	121	57.87 (-0.01)	23.30 (+0.02)	13.54 (+0.00)	5.29 (-0.01)	1.01
2009 (PO 2006 benchmark)	1	615	53.01 (-4.35)	24.94 (+1.23)	14.88 (+1.06)	7.17 (+2.06)	1.33
2011 (7th Award, BE)	1	1,044	50.85 (-0.89)	24.13 (-0.42)	16.09 (+1.47)	8.93 (-0.16)	1.68
2013–2021 (7th Award, actual)	9	2,238	48.07 (-3.67)	25.68 (+1.13)	16.29 (+1.67)	9.96 (+0.87)	1.84
2022–2026 (extended Award, BE)	5	6,809	49.69 (-2.05)	24.91 (+0.36)	16.37 (+1.75)	9.02 (-0.07)	1.54

Notes: Province columns report the mean realized share (%) with the mean deviation from the formula share in parentheses (pp). The final column is the ratio of per-capita transfers in Balochistan to those in Punjab.

Figure 3 traces the deviations year by year. KP's positive deviation is remarkably stable at 1.5 to 1.8 points, consistent with the 1 per cent war-on-terror allocation and straight transfers. Punjab's negative deviation narrowed from almost 6 points in FY2013-14 to about 2 points from FY2021-22 onward, while Sindh's and Balochistan's positive deviations shrank towards zero. The budget years after FY2021-22 are far smoother than the actual years before them, an early indication that budget figures and outturns are generated by different processes.

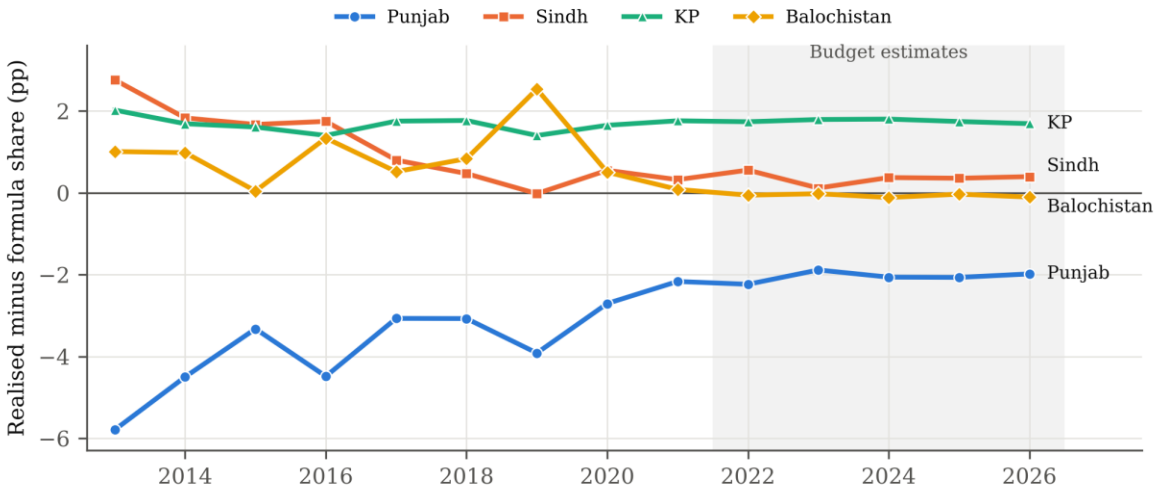


Figure 3. Deviation of realized horizontal shares from formula shares, FY2013-14 to FY2026-27. Source: Author's calculations. Shaded years are budget estimates; earlier years are actual outturns.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Formula Fidelity and Structural Change

The compositional tests in Table 3 confirm H1 sharply. For FY1997-98 to FY2001-02 the mean Aitchison distance between realized and formula compositions is 0.003 and neither the F approximation ($p = 0.312$) nor the exact randomization test ($p = 0.375$) rejects fidelity. For the Seventh Award's actual years the mean distance is more than forty times larger, 0.156, and fidelity is rejected decisively (exact $p = 0.004$, the smallest attainable with nine years being 0.002). Budget years also deviate (F-test $p < 0.001$), though the exact test, limited to 32 sign patterns, gives $p = 0.062$. The sup-F test locates a break in FY2020-21 (sup-F = 10.45, bootstrap $p = 0.044$), after which the mean distance falls from 0.167 to 0.115. The break coincides with the constitution of the Tenth NFC and the switch in the sample from actual to budgeted figures, so it should be read as a change in how allocations are reported and negotiated rather than as a return to the formula: the post-break distance remains far from zero.

Table 3. Compositional Tests of Formula Fidelity (Isometric Log-Ratio Coordinates).

Sample	n (years)	Mean Aitchison distance	Hotelling T^2	F(3, n−3)	p (F)	p (exact sign-flip)
1997–2001 actual	5	0.003	14.1	2.36	0.312	0.375
2013–2021 actual	9	0.156	1,099.9	274.98	<0.001	0.004
2022–2026 BE	5	0.115	11,419.7	1,903.28	<0.001	0.062

Notes: The null hypothesis is that the mean ilr difference between realized and formula compositions is zero. The exact randomization p-value enumerates all 2^n sign flips. Quandt–Andrews sup-F test for a mean shift in the Aitchison distance over FY2013–2026 (15% trimming): sup-F = 10.45, estimated break FY2020–21, bootstrap $p = 0.044$ (Andrews 5% critical value 8.85); mean distance 0.167 before and 0.115 after the break.

Dependence, Persistence and Convergence

Table 4 reports diagnostics for the contiguous block. Residuals from a province-effects regression are strongly cross-sectionally dependent (CD = 6.37, mean absolute correlation 0.72), a natural consequence of all four provinces drawing on the same pool; once year effects are added the dependence disappears (CD = −0.68). The CIPS statistics reject a unit root in log transfers ($p = 0.002$), log per-capita transfers ($p = 0.009$) and log shares ($p = 0.005$) once the common factor is filtered out, and marginally for the formula deviation ($p = 0.063$). The economic reading is that provincial transfers share a single stochastic trend, the size of the divisible pool, around which provincial deviations are mean-reverting. The CCE-MG estimates in Table 5 make the same point: the mean loading on the common transfer factor is 1.000 (s.e. 0.036), while the elasticity of transfers with respect to a province's own population is imprecisely estimated and of no consistent sign.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Table 4. Cross-Sectional Dependence and CIPS Panel Unit-Root Tests, FY2013–2026.

Series	CIPS	10% c.v.	5% c.v.	1% c.v.	Simulated p
ln transfers	-3.801	-2.52	-2.78	-3.26	0.002
ln per-capita transfers	-3.303	-2.52	-2.80	-3.29	0.009
Formula deviation	-2.705	-2.55	-2.77	-3.32	0.063
ln realized share	-3.453	-2.50	-2.74	-3.22	0.005

Notes: CIPS is the cross-sectionally augmented IPS statistic (Pesaran 2007) with intercept; critical values are simulated (3,000 replications) for N = 4 and T = 14 because tabulated values do not cover such a short panel. Pesaran CD test on residuals of a one-way FE convergence regression: CD = 6.37 ($p < 0.001$), mean $|\rho| = 0.72$; after adding year effects: CD = -0.68 ($p = 0.499$).

The TWFE convergence coefficient in Table 5 is −0.479, significant with conventional standard errors but much less so under Driscoll–Kraay errors and the wild cluster bootstrap ($p = 0.061$). The half-panel jackknife correction reduces it to −0.095, implying a half-life of about seven years. Convergence is therefore present but slow, and much of the apparent speed in the uncorrected estimate is Nickell bias. Figure 4 shows the corresponding σ-convergence. The Theil index of per-capita transfers was close to zero under the 1997 Award, rose by two orders of magnitude with the Seventh Award as Balochistan's per-capita receipts rose to more than twice Punjab's, peaked in FY2013-14 and FY2019-20, and has since declined steadily (slope −0.0011 per year, $t = -4.41$), with the max–min ratio falling from 2.06 in FY2013-14 to 1.52 in FY2026-27. H3 is supported: the Seventh Award produced a step increase in equalization towards the least populous province, which is being slowly diluted as the pool grows and fixed add-ons shrink in relative terms.

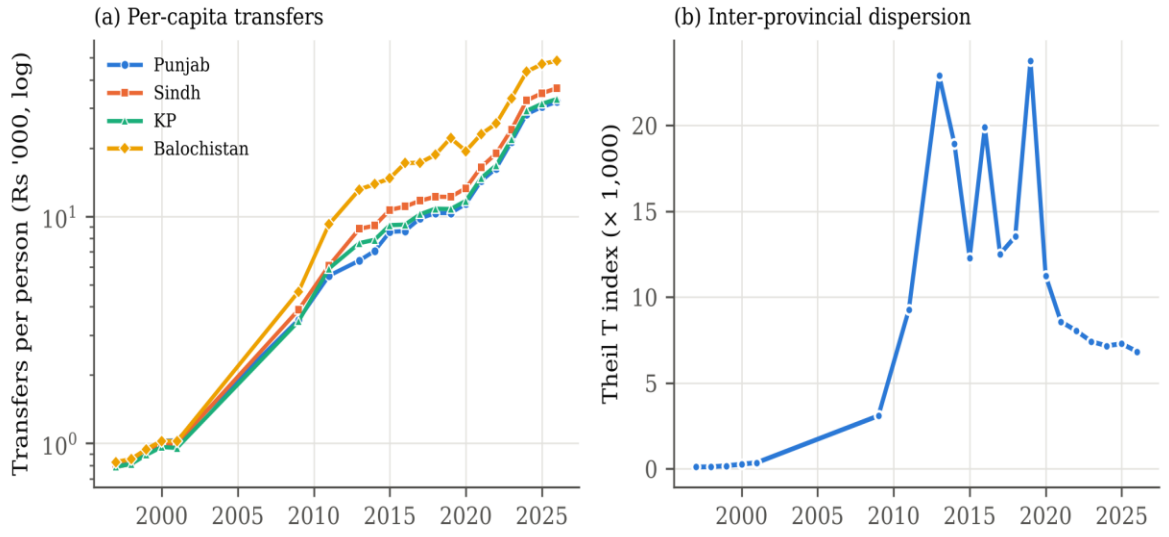


Figure 4. Per-capita transfers by province and inter-provincial dispersion, FY1997-98 to FY2026-27. Source: Author's calculations. Panel (b) is the population-weighted Theil T index of per-capita transfers.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Table 5. Panel Estimates: Convergence, Census Responsiveness and Discretionary Grants.

Parameter	Estimate	CR1 s.e.	Driscoll–Kraay s.e.	WCR bootstrap p	Obs.
(1) β-convergence, Δln pc on ln pc(t−1), TWFE	-0.479	0.048	0.158	0.061	52
(1a) Half-panel jackknife bias-corrected β	-0.095	–	–	–	52
(2) Census responsiveness θ, ln s on ln pop share, TWFE	-1.256	0.644	0.350	0.332 (H_0: θ = 0)	56
(2a) Same, H_0: θ = formula-implied 0.78				0.302	
(3) KP × FATA merger effect on grants (% of transfers), TWFE	25.42	0.83	–	0.058	32
(4) CCE-MG elasticity of transfers to own population	-2.15	3.00 (MG)	–	–	56
(4a) CCE-MG loading on common transfer factor	1.000	0.036 (MG)	–	–	56

Notes: TWFE = province and year fixed effects. With only four clusters, conventional cluster-robust inference is unreliable; the restricted wild cluster (WCR) bootstrap uses Webb six-point weights and enumerates all 6^4 = 1,296 draws, so the smallest attainable p-value is 1/1,296. Driscoll–Kraay standard errors use a Bartlett kernel with two lags. Half-life of deviations implied by the jackknife estimate: 7.0 years.

The census-rigidity hypothesis (H2) receives qualified support. The estimated responsiveness of shares to current population is negative (θ = −1.26) and not significantly different from zero (bootstrap p = 0.332). It is also not statistically distinguishable from the formula-implied 0.78 (p = 0.302), so the test has little power: with four provinces and smooth interpolated populations, demographic variation is too limited to identify θ precisely. The structural evidence is more telling. Re-running the formula with 2023 census shares while holding the weights constant would cut Punjab's share by 3.25 points and raise KP's by 2.68 and Balochistan's by 0.92 points (Table 10). Applied to the FY2026-27 budget, the frozen census base keeps about Rs 288 billion with Punjab and withholds about Rs 237 billion from KP and Rs 81 billion from Balochistan relative to a census-updated formula.

Discretionary grants tell a complementary story. Federal loans and grants outside the pool averaged 15.5 per cent of NFC transfers for KP over FY2013–2020 against 3.4 per cent for Punjab. After the FATA merger, KP's grants rose by 25.4 percentage points of its transfers relative to other provinces (bootstrap p = 0.058), and across province-years grant intensity is positively correlated with the formula deviation (Spearman ρ = 0.35, p = 0.051). The federation has responded to the merger through discretionary channels rather than through the formula, as bargaining theories of transfers predict (Dixit and Londregan 1998; Rodden 2006).

Credibility of Ex-Ante Figures

Table 6 and Figure 5 bear out H4. The projections of the 1996 Commission overstated realized transfers by 49 log points on average, with the shortfall widening from 33 log points in FY1997-98 to 67 in FY2001-02. Rationality is rejected (Wald = 886.1, bootstrap p = 0.019), and 95 per cent of the mean squared error is bias. Because the shortfall was nearly identical across provinces, it

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

was a vertical failure of revenue projection, not a horizontal failure of the formula: the 1997 Award distributed faithfully a pool that never materialized. Under the Seventh Award, by contrast, budget estimates are unbiased predictors of outturns. The mean error is −0.6 log points, the RMSE is 4.0, rationality is not rejected ($p = 0.785$) and 94 per cent of the residual error is unsystematic. Fiscal contestation since 2010 has therefore centred less on the accuracy of the numbers than on how the pool is to be shared and topped up.

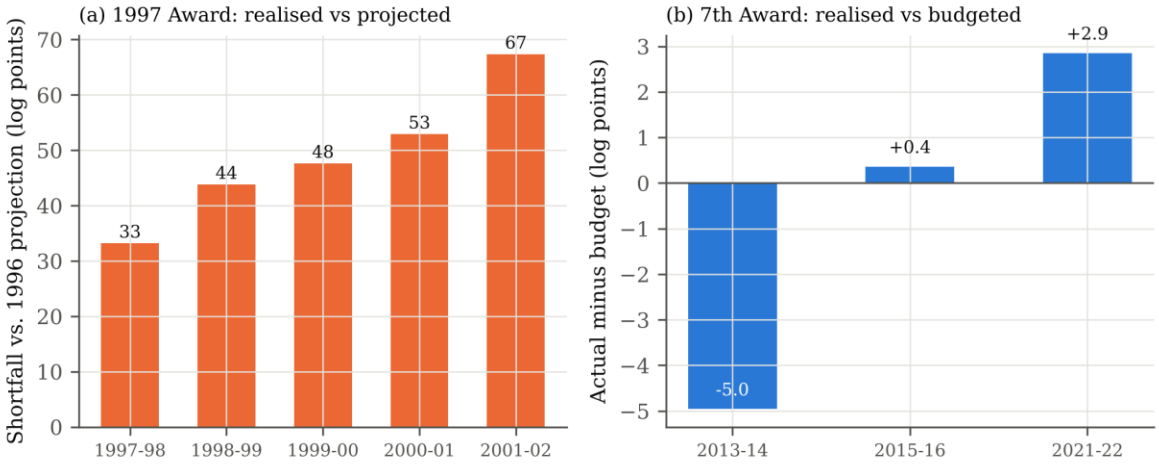


Figure 5. Ex-ante projections and realized transfers.
Source: Author's calculations. Errors are 100 × ln(actual/ex-ante), averaged across provinces.

Table 6. Budget Credibility: Mincer–Zarnowitz Rationality Tests and Theil Decomposition.

Ex-ante benchmark	n	Mean error (log pts)	RMSE (log pts)	α (s.e.)	β (s.e.)	Wald α=0, β=1 [WCR p]	Theil U^M / U^s / U^c
Award projection 1997–2001	20	-49.0	50.3	-0.375 (0.060)	0.967 (0.016)	886.1 [0.019]	0.95 / 0.00 / 0.05
Budget estimate 2013, 2015, 2021	12	-0.6	4.0	-0.069 (0.087)	1.010 (0.014)	0.7 [0.785]	0.02 / 0.04 / 0.94

Notes: Error = 100 × ln(actual/ex-ante). The Mincer–Zarnowitz regression is $\ln A = \alpha + \beta \ln F + u$. U^M, U^s and U^c are the bias, variance and covariance proportions of the mean squared error and sum to one.

The Revealed Formula

The inverse-formula network recovers the weights that would reproduce the mean realized allocation of FY2013–2021 almost exactly (fit RMSE 0.002 points). Table 7 shows that these revealed weights differ strikingly from the statutory ones. The realized allocation is consistent with a population weight of 0.682 (90 per cent interval 0.654 to 0.710) rather than 0.82, a poverty and backwardness weight of 0.231 rather than 0.103, and a revenue weight of 0.081 rather than 0.05, with almost no weight on inverse density. In other words, the Seventh Award in operation has been considerably more needs-based and less population-based than the Award on paper. This is not because the formula has been set aside but

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

because the add-ons, straight transfers of natural resource royalties and the war-on-terror allocation all flow disproportionately to the less populous provinces, and their combined effect mimics a formula with heavier weight on poverty and revenue generation. In the budget years from FY2022-23 the revealed population weight rises to 0.748, which is consistent with the narrowing of deviations in Figure 3.

Table 7. Revealed Formula Weights Learned by the Softmax Inverse-Formula Network.

Criterion	Official 7th Award weight	Revealed weight, actuals FY2013–21	90% bootstrap interval	Revealed weight, budget FY2022–26
Population	0.820	0.682	[0.654, 0.710]	0.748
Poverty / backwardness	0.103	0.231	[0.218, 0.242]	0.204
Revenue collection / generation	0.050	0.081	[0.065, 0.100]	0.048
Inverse population density	0.027	0.006	[0.003, 0.012]	0.001

Notes: The network maps the fixed 7th Award indicator matrix to realized mean shares through a single softmax-parameterized layer trained by Adam, so the learned weights are non-negative and sum to one. Fit RMSE: 0.002 pp (actuals) and 0.087 pp (budget). The interval is from 300 year-block bootstrap resamples.

Deep Learning Forecasts and their Interpretation

Table 8 and Figure 6 report the forecasting contest. The historical mean is the most accurate single model (RMSE 11.57 points of growth), followed by the deep MLP ensemble (12.73) and the naive forecast (13.30). The ensemble beats the naive forecast, ridge regression, random forests and gradient boosting on overall RMSE, but none of these differences is statistically significant (DM-HLN $p = 0.664$ against naive). The LSTM is the most accurate model over the actual-outturn years FY2019-20 to FY2021-22 (RMSE 9.24 against 11.45 for the historical mean), yet it deteriorates in the budget years, where definitional changes in the reported series dominate. Only pooled OLS with all covariates is excluded from the 90 per cent model confidence set; every other model, including all three neural architectures, is statistically indistinguishable from the best. Hyperparameter selection is informative in itself: the validation step chose heavily penalized networks (L2 penalties of 10 or 50) at six of the eight origins, and a preliminary specification with a fixed, lighter penalty performed markedly worse (RMSE 19.07), which shows how sensitive deep networks are to regularization in a panel of this size. H5 is supported.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

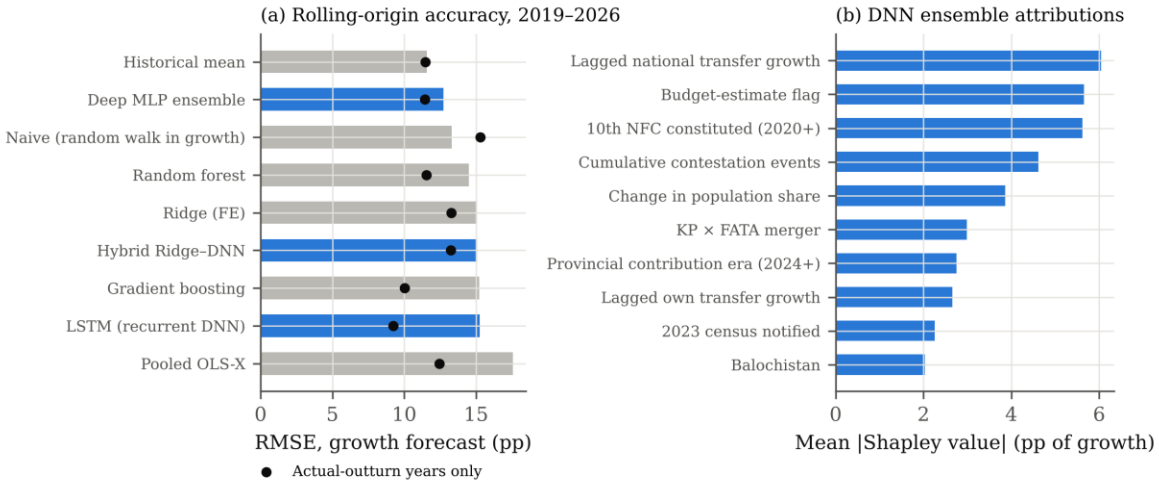


Figure 6. Forecast accuracy of econometric and deep learning models and Shapley attributions of the deep ensemble. Source: Author's calculations. Blue bars denote neural network models; dots show RMSE over actual-outturn years only.

Table 8. Out-of-Sample One-Year-Ahead Forecast Accuracy for Provincial Transfer Growth, FY2019–2026.

Model	RMSE (pp)	MAE (pp)	RMSE, actual years	RMSE, budget years	DM-HLN vs naive [p]	In 90% MCS
Historical mean	11.57	9.99	11.45	11.64	-0.97 [0.341]	✓
Deep MLP ensemble	12.73	11.21	11.45	13.45	-0.44 [0.664]	✓
Naive (random walk in growth)	13.30	10.77	15.31	11.94	–	✓
Random forest	14.50	13.30	11.54	16.02	0.81 [0.422]	✓
Ridge (FE)	15.09	13.79	13.26	16.08	0.97 [0.340]	✓
Hybrid Ridge–DNN	15.09	13.78	13.25	16.09	0.97 [0.341]	✓
Gradient boosting	15.23	13.66	10.02	17.63	1.14 [0.263]	✓
LSTM (recurrent DNN)	15.24	12.52	9.24	17.90	0.79 [0.435]	✓
Pooled OLS-X	17.58	15.61	12.46	20.04	1.91 [0.066]	–

Notes: Rolling-origin design with 32 forecasts (4 provinces × 8 origins); every model is re-estimated at each origin using only earlier data, and DNN hyperparameters are selected on the last year of each training window. DM-HLN = Diebold–Mariano test with the Harvey–Leybourne–Newbold small-sample correction; positive values indicate a larger squared loss than the naive forecast. MCS = model confidence set (Hansen, Lunde and Nason 2011), T_max statistic, block bootstrap with 2,000 replications.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

The Shapley attributions in Figure 6(b) explain why. The largest contributions to the ensemble's predictions come from lagged national transfer growth (mean absolute Shapley value 6.0 points), the budget-estimate flag (5.7), the indicator for the Tenth Commission era (5.6) and the cumulative count of contestation events (4.6). Province-specific economic variables, such as lagged formula deviations or relative per-capita receipts, contribute about one point each. The network has learned that provincial transfer growth is driven by the common pool and by the regime and reporting environment, which is exactly what the econometric diagnostics found. Its full-sample fit ($R^2 = 0.98$) contrasts sharply with its out-of-sample accuracy, a warning that in-sample performance of deep models on panels of this size says little about their predictive value.

Double Machine Learning

Table 9 reports the convergence coefficient estimated by double/debiased machine learning. With deep network nuisance functions the estimate is −0.242 (s.e. 0.032); with random forests −0.144 and with ridge regression −0.298. All three are negative and lie between the TWFE estimate (−0.479) and the jackknife estimate (−0.095), which locates the range of plausible convergence speeds and confirms that convergence of per-capita transfers is real but moderate. The dependence of the point estimate on the learner is itself a finding: with 48 observations, the choice of nuisance model shifts the estimate by a factor of two, and the conventional standard errors understate the true uncertainty.

Table 9. Double/Debiased Machine Learning Estimates of the Convergence Coefficient.

Nuisance learner	θ (Δln pc on ln pc(t−1))	Cluster s.e.	95% interval
Deep neural network (32-16, 5-seed ensemble)	-0.242	0.032	[-0.306, -0.179]
Random forest	-0.144	0.008	[-0.160, -0.128]
Ridge regression	-0.298	0.016	[-0.330, -0.266]

Notes: Partially linear model with four year-block cross-fitting folds (n = 48). Controls: lagged national growth, lagged formula deviation, budget flag, event indicators, cumulative contestations and province indicators. Standard errors are clustered by province and should be read as indicative given four clusters.

Formula Counterfactuals for the Tenth NFC

Table 10 and Figure 7 turn to the future. Updating population to the 2023 census with unchanged weights reduces Punjab's share by 3.25 points and Sindh's by 0.34 points and raises KP's by 2.68 points and Balochistan's by 0.92 points. Weight uncertainty does not alter the direction of these changes for Punjab and KP: across 20,000 Dirichlet draws Punjab never gains and KP always gains, whereas Balochistan gains in 84 per cent of draws and Sindh in 21 per cent.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

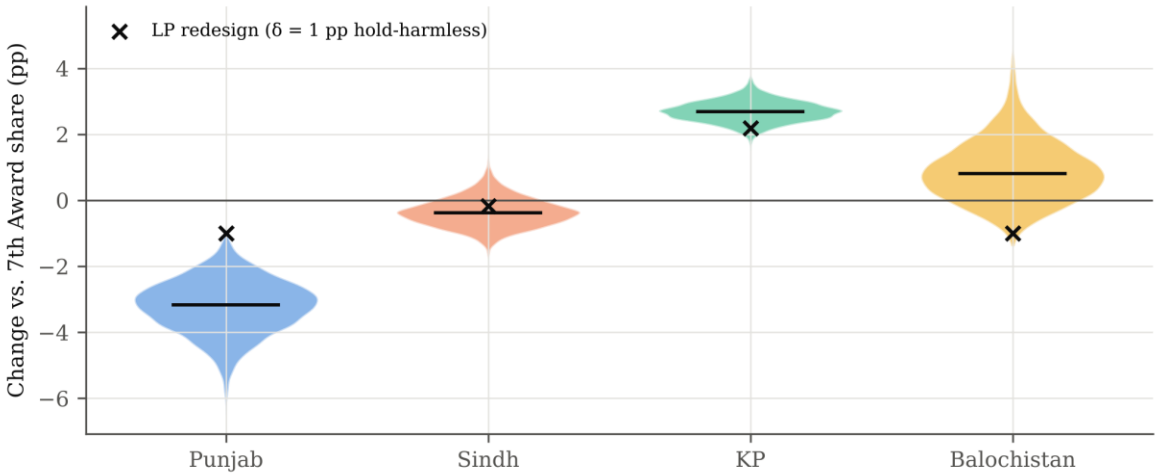


Figure 7. Distribution of horizontal share changes under the 2023 census and uncertain weights, with the constrained linear-programming redesign. Source: Author's calculations from 20,000 Dirichlet draws of formula weights (κ = 200).

Table 10. Formula Counterfactuals for the Tenth NFC: Census Updating, Weight Uncertainty and Constrained Redesign.

Province	7th Award share (%)	2023 census, same weights (Δpp)	Dirichlet 90% interval	P(gain)	LP redesign δ = 1 pp (Δpp)	Shapley: census / reweighting (pp)	Census effect on FY2026-27 pool (Rs bn)
Punjab	51.74	48.49 (-3.25)	[47.04, 49.76]	0.00	50.74 (-1.00)	-3.39 / +2.39	-288
Sindh	24.55	24.21 (-0.34)	[23.51, 25.00]	0.21	24.37 (-0.18)	-0.36 / +0.18	-30
KP	14.62	17.30 (+2.68)	[16.77, 17.84]	1.00	16.80 (+2.18)	+2.79 / -0.61	+237
Balochistan	9.09	10.01 (+0.92)	[8.63, 11.73]	0.84	8.09 (-1.00)	+0.95 / -1.95	+81

Notes: Dirichlet draws (20,000) centre on the official weights with concentration κ = 200. The LP redesign maximizes the combined weight on poverty and inverse density subject to a population weight ≥ 0.70, a revenue weight ≥ 0.05 and no province losing more than δ pp relative to its 7th Award share; optimal weights are population 0.888, poverty 0.046, revenue 0.050, inverse density 0.016. No feasible solution exists for δ = 0.5 pp. The final column applies the census-only change to the FY2026-27 budgeted total of Rs 8,847 billion.

The linear programme exposes a redistributive trilemma. A hold-harmless rule limiting any province's loss to half a percentage point is infeasible: with the 2023 census, no admissible

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

weights keep both Punjab and Balochistan within that margin, because Punjab loses through the population update while Balochistan's gain depends on the very equalizing criteria that would have to be raised. The tightest feasible guarantee, one point, is achieved only by raising the population weight to 0.888 and cutting poverty and density weights to 0.046 and 0.016. Under that design KP gains 2.18 points while Punjab and Balochistan each lose exactly one. The Shapley decomposition shows the mechanism: the census update alone would move Punjab by −3.39 and KP by +2.79 points, and the reweighting needed to protect Punjab gives back +2.39 points to Punjab at the cost of −1.95 points to Balochistan. In other words, a formula that respects demographic change, protects the largest province and preserves equalization for the smallest cannot be achieved simultaneously within a fixed provincial pool. Bootstrapping the historical growth of the pool gives a median provincial pool of about Rs 13.5 trillion in FY2029-30 (80 per cent interval Rs 11.2 to 16.6 trillion), in which the census update alone would be worth about Rs 440 billion to Punjab and Rs 360 billion to KP each year.

Discussion: The Three Contestations Revisited

The Constitutional Contestation

Article 160(3A) has done what it was designed to do. The vertical share has not fallen below 57.5 per cent in any year since FY2011-12, and the budgeted pool has grown more than eightfold. The evidence shows, however, that the constitutional floor protects an aggregate rather than an allocation. Horizontal shares have deviated persistently from the formula, discretionary grants have absorbed the fiscal consequences of the FATA merger, and since 2020 the Centre has secured contributions from provinces through agreements and Article 164 grants that leave the formula share untouched while reducing effective provincial resources. The constitutional contestation has therefore migrated from the vertical share, which is entrenched, to instruments around it, which are not. This mirrors comparative experience in which entrenched fiscal rules induce workarounds rather than compliance (Rodden 2006; Watts 2008), and it suggests that the durability of the 2010 settlement depends on how these side instruments are governed. A Tenth Award that writes provincial contributions to national functions into the Award itself, with explicit rules and sunset clauses, would be more consistent with Article 160(3A) than recurrent ad hoc arrangements.

The Fiscal Contestation

Two distinct fiscal problems emerge from the analysis. The first, the systematic overstatement of the pool at Award time, was the defining failure of the 1997 Award: its projections were almost pure bias, and provinces bore the full burden of a revenue shortfall they did not control. The Seventh Award's budget estimates, by contrast, pass rationality tests, so the federal-provincial dispute since 2010 has not been about forecasting. The second problem is the vertical imbalance created by a fixed and large provincial share combined with expanding federal obligations, which is the origin of the contribution and Article 164 arrangements. The CCE-MG and CIPS evidence that all provinces load one-for-one on a common transfer factor underlines that provincial budgets are hostage to federal revenue performance. A Tenth Award that couples the vertical share to explicit federal expenditure responsibilities, and that includes a stabilization mechanism smoothing provincial receipts over the revenue cycle, would address the actual sources of fiscal contestation better than renegotiating the headline share.

The Administrative Contestation

The administrative contestation is the most acute. The formula still relies on the 1998 census, and the revealed-weight analysis shows that the allocation in practice already departs from it

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

through channels that are opaque, weakly rule-bound and difficult to audit. Updating to the 2023 census would shift about 3.25 points of the pool, equivalent to roughly Rs 290 billion in FY2026-27, away from Punjab. The trilemma result implies that this adjustment cannot be made painless by tuning weights within a fixed pool. Three implications follow. First, the transition will need to be phased, for example by moving population shares towards the latest census over the term of the Award. Second, the add-ons and straight transfers that currently tilt the allocation towards need should be made explicit and formula-based rather than left as side provisions, so that the revealed and statutory formulas converge. Third, the merger of the former tribal areas should be financed through a transparent, time-bound component of the Award rather than through discretionary grants, whose growth after 2018 is the clearest evidence of administrative adjustment outside the formula. Each of these steps would also strengthen the link between provincial resources and the delivery of devolved functions at the local level that Article 140A envisages (Cheema, Khan and Myerson 2015).

What Deep Learning Can and Cannot Do for Fiscal Federalism

The deep learning results carry a methodological lesson for the study of federal finance. Neural networks are attractive because transfer systems are non-linear and regime-dependent, but in a federation with four or five constituent units the data cannot sustain models with many parameters. Here, the networks became competitive only when heavily regularized and selected without look-ahead, and even then they were statistically indistinguishable from the historical mean. Their most useful contributions were structural rather than predictive: the softmax inverse-formula layer recovered an interpretable 'revealed formula' that conventional regression could not express, Shapley attributions confirmed that common-pool and regime variables dominate provincial transfer dynamics, and double machine learning bracketed the plausible range of convergence speeds. Researchers applying deep learning to federal systems with few units should treat these models as tools for structured interpretation and robustness, report honest out-of-sample comparisons against simple benchmarks, and use small-cluster inference throughout.

Conclusion

This article has examined the operation of Pakistan's NFC Award with an original, fully sourced dataset and an integrated econometric and deep learning framework designed for a small federation. Five findings stand out. First, the formula governed horizontal allocation exactly under the 1997 Award but not under the Seventh Award, where realized shares deviate persistently and the realized allocation behaves as if population carried a weight of about two-thirds rather than 82 per cent. Second, the formula's frozen census base has created a growing gap between applied and demographically updated shares, worth about Rs 290 billion to Punjab and Rs 240 billion against KP in FY2026-27. Third, the Seventh Award produced a step increase in equalization of per-capita transfers that is slowly converging, with a bias-corrected half-life of about seven years. Fourth, ex-ante projections embedded in the 1997 Award were almost pure bias, whereas Seventh Award budget estimates are rational predictors. Fifth, regularized deep neural networks and LSTM models forecast transfer growth about as well as simple benchmarks but not better, and their main value lies in interpretation rather than prediction.

For the Tenth NFC these findings imply a reform agenda centred on transparency and phasing rather than on the headline vertical share: incorporate add-ons and straight transfers into an explicit formula, phase in the 2023 census, finance the merged districts through a time-bound Award component, and codify provincial contributions to national functions within the Award. The study

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

has limitations. The panel is short and contains gaps; budget figures mix definitions over time; the population series relies on interpolation; and the counterfactuals hold non-population indicators at their 2010 values. Future research should extend the panel with monthly federal fiscal-operations data and district-level accounts, which would allow richer causal designs and give deep learning models the sample size they require. Pakistan's federal bargain has been resilient because it has been renegotiated; the evidence here suggests that the next renegotiation will be more durable if the formula on paper and the formula in practice are brought back into line.

Notes

1. Fiscal years run from 1 July to 30 June and are denoted by the calendar year in which they begin when abbreviated in tables (e.g. 2013 for FY2013-14).
2. The 1991 Award's 80 per cent share applied to a narrower pool than the 1997 and later Awards, so the fall to 37.5 per cent in 1997 overstates the reduction in provincial resources.
3. Exact randomization p-values are bounded below by $1/2^n$ (n years) and the enumerated wild cluster bootstrap p-values by 1/1,296; all reported p-values should be read with these resolutions in mind.
4. Replication code (Python) reproduces every table and figure from the dataset workbook. The LSTM, the inverse-formula network, the wild cluster bootstrap, CIPS critical values and the model confidence set are implemented directly in NumPy and SciPy; feed-forward networks, random forests and gradient boosting use scikit-learn.
5.

REFERENCES

Adeney, Katharine. 2007. Federalism and Ethnic Conflict Regulation in India and Pakistan. New York: Palgrave Macmillan.

Adeney, Katharine. 2012. "A Step Towards Inclusive Federalism in Pakistan? The Politics of the 18th Amendment." Publius: The Journal of Federalism 42 (4): 539–565.

Ahmad, Iftikhar, Usman Mustafa, and Mahmood Khalid. 2007. "National Finance Commission Awards in Pakistan: A Historical Perspective." PIDE Working Papers 2007:33. Islamabad: Pakistan Institute of Development Economics.

Aitchison, John. 1986. The Statistical Analysis of Compositional Data. London: Chapman and Hall.

Andrews, Donald W. K. 1993. "Tests for Parameter Instability and Structural Change with Unknown Change Point." Econometrica 61 (4): 821–856.

Athey, Susan, and Guido W. Imbens. 2019. "Machine Learning Methods That Economists Should Know About." Annual Review of Economics 11: 685–725.

Bahl, Roy. 2000. "Intergovernmental Transfers in Developing and Transition Countries: Principles and Practice." Municipal Finance Background Series. Washington, DC: World Bank.

Bahl, Roy, Sally Wallace, and Musharraf Cyan. 2008. "Pakistan: Provincial Government Taxation." International Studies Program Working Paper 08-12. Atlanta: Andrew Young School of Policy Studies, Georgia State University.

Barro, Robert J., and Xavier Sala-i-Martin. 1992. "Convergence." Journal of Political Economy 100 (2): 223–251.

Bird, Richard M., and Michael Smart. 2002. "Intergovernmental Fiscal Transfers: International Lessons for Developing Countries." World Development 30 (6): 899–912.

Boadway, Robin, and Anwar Shah, eds. 2007. Intergovernmental Fiscal Transfers: Principles and Practice. Washington, DC: World Bank.

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Boadway, Robin, and Anwar Shah. 2009. Fiscal Federalism: Principles and Practice of Multiorder Governance. Cambridge: Cambridge University Press.

Breiman, Leo. 2001. "Random Forests." Machine Learning 45 (1): 5–32.

Cameron, A. Colin, Jonah B. Gelbach, and Douglas L. Miller. 2008. "Bootstrap-Based Improvements for Inference with Clustered Errors." Review of Economics and Statistics 90 (3): 414–427.

Cheema, Ali, Adnan Q. Khan, and Roger B. Myerson. 2015. "Breaking the Countercyclical Pattern of Local Democracy in Pakistan." In Is Decentralization Good for Development? Perspectives from Academics and Policy Makers, edited by Jean-Paul Faguet and Caroline Pöschl, 68–79. Oxford: Oxford University Press.

Cheema, Ali, Asim Ijaz Khwaja, and Adnan Qadir. 2006. "Local Government Reforms in Pakistan: Context, Content and Causes." In Decentralization and Local Governance in Developing Countries: A Comparative Perspective, edited by Pranab Bardhan and Dilip Mookherjee, 257–284. Cambridge, MA: MIT Press.

Chernozhukov, Victor, Denis Chetverikov, Mert Demirer, Esther Duflo, Christian Hansen, Whitney Newey, and James Robins. 2018. "Double/Debiased Machine Learning for Treatment and Structural Parameters." Econometrics Journal 21 (1): C1–C68.

Chudik, Alexander, and M. Hashem Pesaran. 2015. "Common Correlated Effects Estimation of Heterogeneous Dynamic Panel Data Models with Weakly Exogenous Regressors." Journal of Econometrics 188 (2): 393–420.

Dhaene, Geert, and Koen Jochmans. 2015. "Split-Panel Jackknife Estimation of Fixed-Effect Models." Review of Economic Studies 82 (3): 991–1030.

Diebold, Francis X., and Roberto S. Mariano. 1995. "Comparing Predictive Accuracy." Journal of Business & Economic Statistics 13 (3): 253–263.

Dixit, Avinash, and John Londregan. 1998. "Fiscal Federalism and Redistributive Politics." Journal of Public Economics 68 (2): 153–180.

Driscoll, John C., and Aart C. Kraay. 1998. "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data." Review of Economics and Statistics 80 (4): 549–560.

Egozcue, Juan José, Vera Pawlowsky-Glahn, Glòria Mateu-Figueras, and Carles Barceló-Vidal. 2003. "Isometric Logratio Transformations for Compositional Data Analysis." Mathematical Geology 35 (3): 279–300.

Elazar, Daniel J. 1987. Exploring Federalism. Tuscaloosa: University of Alabama Press.

Farrell, Max H., Tengyuan Liang, and Sanjog Misra. 2021. "Deep Neural Networks for Estimation and Inference." Econometrica 89 (1): 181–213.

Goodfellow, Ian, Yoshua Bengio, and Aaron Courville. 2016. Deep Learning. Cambridge, MA: MIT Press.

Government of Pakistan. 1973. The Constitution of the Islamic Republic of Pakistan. Islamabad: National Assembly of Pakistan.

Government of Pakistan. 2010a. The Constitution (Eighteenth Amendment) Act, 2010. Islamabad: The Gazette of Pakistan.

Government of Pakistan. 2010b. Distribution of Revenues and Grants-in-Aid Order, 2010 (President's Order No. 5 of 2010). Islamabad: The Gazette of Pakistan.

Government of Pakistan, Finance Division. 2009. Report of the National Finance Commission 2009. Islamabad: Finance Division.

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Government of Pakistan, Finance Division. 2024. Pakistan Economic Survey 2023–24. Islamabad: Finance Division.

Government of Pakistan, Finance Division. 2025a. Pakistan Economic Survey 2024–25. Islamabad: Finance Division.

Government of Pakistan, Finance Division. 2025b. Federal Budget 2025–26: Budget in Brief. Islamabad: Finance Division.

Gramlich, Edward M. 1977. "Intergovernmental Grants: A Review of the Empirical Literature." In The Political Economy of Fiscal Federalism, edited by Wallace E. Oates, 219–239. Lexington, MA: Lexington Books.

Hansen, Peter R., Asger Lunde, and James M. Nason. 2011. "The Model Confidence Set." Econometrica 79 (2): 453–497.

Harvey, David, Stephen Leybourne, and Paul Newbold. 1997. "Testing the Equality of Prediction Mean Squared Errors." International Journal of Forecasting 13 (2): 281–291.

Hines, James R., and Richard H. Thaler. 1995. "Anomalies: The Flypaper Effect." Journal of Economic Perspectives 9 (4): 217–226.

Hochreiter, Sepp, and Jürgen Schmidhuber. 1997. "Long Short-Term Memory." Neural Computation 9 (8): 1735–1780.

IMF (International Monetary Fund). 2019. Pakistan: Request for an Extended Arrangement under the Extended Fund Facility. IMF Country Report No. 19/212. Washington, DC: IMF.

Inman, Robert P. 2008. "The Flypaper Effect." NBER Working Paper No. 14579. Cambridge, MA: National Bureau of Economic Research.

Jalal, Ayesha. 1995. Democracy and Authoritarianism in South Asia: A Comparative and Historical Perspective. Cambridge: Cambridge University Press.

Khemani, Stuti. 2007. "Does Delegation of Fiscal Policy to an Independent Agency Make a Difference? Evidence from Intergovernmental Transfers in India." Journal of Development Economics 82 (2): 464–484.

Kingma, Diederik P., and Jimmy Ba. 2015. "Adam: A Method for Stochastic Optimization." Paper presented at the 3rd International Conference on Learning Representations (ICLR), San Diego.

Kornai, János. 1986. "The Soft Budget Constraint." Kyklos 39 (1): 3–30.

Kornai, János, Eric Maskin, and Gérard Roland. 2003. "Understanding the Soft Budget Constraint." Journal of Economic Literature 41 (4): 1095–1136.

Lakshminarayanan, Balaji, Alexander Pritzel, and Charles Blundell. 2017. "Simple and Scalable Predictive Uncertainty Estimation Using Deep Ensembles." In Advances in Neural Information Processing Systems 30, 6402–6413.

LeCun, Yann, Yoshua Bengio, and Geoffrey Hinton. 2015. "Deep Learning." Nature 521 (7553): 436–444.

Lundberg, Scott M., and Su-In Lee. 2017. "A Unified Approach to Interpreting Model Predictions." In Advances in Neural Information Processing Systems 30, 4765–4774.

MacKinnon, James G., and Matthew D. Webb. 2017. "Wild Bootstrap Inference for Wildly Different Cluster Sizes." Journal of Applied Econometrics 32 (2): 233–254.

Mincer, Jacob A., and Victor Zarnowitz. 1969. "The Evaluation of Economic Forecasts." In Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance, edited by Jacob A. Mincer, 3–46. New York: National Bureau of Economic Research.

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Mullainathan, Sendhil, and Jann Spiess. 2017. "Machine Learning: An Applied Econometric Approach." Journal of Economic Perspectives 31 (2): 87–106.

Musgrave, Richard A. 1959. The Theory of Public Finance: A Study in Public Economy. New York: McGraw-Hill.

Mustafa, Usman. 2011. "Fiscal Federalism in Pakistan: The 7th National Finance Commission Award and Its Implications." PIDE Working Papers 2011:73. Islamabad: Pakistan Institute of Development Economics.

Nickell, Stephen. 1981. "Biases in Dynamic Models with Fixed Effects." Econometrica 49 (6): 1417–1426.

Oates, Wallace E. 1972. Fiscal Federalism. New York: Harcourt Brace Jovanovich.

Oates, Wallace E. 1999. "An Essay on Fiscal Federalism." Journal of Economic Literature 37 (3): 1120–1149.

Oates, Wallace E. 2005. "Toward a Second-Generation Theory of Fiscal Federalism." International Tax and Public Finance 12 (4): 349–373.

Pakistan Bureau of Statistics. 2023. Population and Housing Census 2023: Provisional Results. Islamabad: Pakistan Bureau of Statistics.

Pesaran, M. Hashem. 2006. "Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure." Econometrica 74 (4): 967–1012.

Pesaran, M. Hashem. 2007. "A Simple Panel Unit Root Test in the Presence of Cross-Section Dependence." Journal of Applied Econometrics 22 (2): 265–312.

Pesaran, M. Hashem. 2015. "Testing Weak Cross-Sectional Dependence in Large Panels." Econometric Reviews 34 (6–10): 1089–1117.

Qian, Yingyi, and Barry R. Weingast. 1997. "Federalism as a Commitment to Preserving Market Incentives." Journal of Economic Perspectives 11 (4): 83–92.

Quandt, Richard E. 1960. "Tests of the Hypothesis That a Linear Regression System Obeys Two Separate Regimes." Journal of the American Statistical Association 55 (290): 324–330.

Rabbani, Mian Raza. 2012. A Biography of Pakistani Federalism: Unity in Diversity. Islamabad: Leo Books.

Rao, M. Govinda, and Nirvikar Singh. 2005. The Political Economy of Federalism in India. New Delhi: Oxford University Press.

Riker, William H. 1964. Federalism: Origin, Operation, Significance. Boston: Little, Brown.

Rodden, Jonathan. 2002. "The Dilemma of Fiscal Federalism: Grants and Fiscal Performance around the World." American Journal of Political Science 46 (3): 670–687.

Rodden, Jonathan. 2006. Hamilton's Paradox: The Promise and Peril of Fiscal Federalism. Cambridge: Cambridge University Press.

Rodden, Jonathan, Gunnar S. Eskeland, and Jennie Litvack, eds. 2003. Fiscal Decentralization and the Challenge of Hard Budget Constraints. Cambridge, MA: MIT Press.

Sabir, Muhammad. 2010. "Financial Implications of the 7th NFC Award and the Impact on Social Services." Pakistan Development Review 49 (4): 387–403.

Shah, Anwar. 2012. "The 18th Constitutional Amendment: Glue or Solvent for Nation Building and Citizenship in Pakistan?" Lahore Journal of Economics 17 (Special Edition): 387–424.

Shapley, Lloyd S. 1953. "A Value for n-Person Games." In Contributions to the Theory of Games, Vol. II, edited by H. W. Kuhn and A. W. Tucker, 307–317. Princeton: Princeton University Press.

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Shorrocks, Anthony F. 2013. "Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value." Journal of Economic Inequality 11 (1): 99–126.

Stepan, Alfred. 1999. "Federalism and Democracy: Beyond the U.S. Model." Journal of Democracy 10 (4): 19–34.

Štrumbelj, Erik, and Igor Kononenko. 2014. "Explaining Prediction Models and Individual Predictions with Feature Contributions." Knowledge and Information Systems 41 (3): 647–665.

Talbot, Ian. 2009. Pakistan: A Modern History. 3rd ed. London: Hurst.

Theil, Henri. 1966. Applied Economic Forecasting. Amsterdam: North-Holland.

Tiebout, Charles M. 1956. "A Pure Theory of Local Expenditures." Journal of Political Economy 64 (5): 416–424.

Treisman, Daniel. 2007. The Architecture of Government: Rethinking Political Decentralization. Cambridge: Cambridge University Press.

Varian, Hal R. 2014. "Big Data: New Tricks for Econometrics." Journal of Economic Perspectives 28 (2): 3–28.

Waseem, Mohammad. 2010. Federalism in Pakistan. Ottawa: Forum of Federations.

Watts, Ronald L. 2008. Comparing Federal Systems. 3rd ed. Montreal and Kingston: McGill-Queen's University Press.

Webb, Matthew D. 2023. "Reworking Wild Bootstrap-Based Inference for Clustered Errors." Canadian Journal of Economics 56 (3): 839–858.

Weingast, Barry R. 1995. "The Economic Role of Political Institutions: Market-Preserving Federalism and Economic Development." Journal of Law, Economics, & Organization 11 (1): 1–31.

Weingast, Barry R. 2009. "Second Generation Fiscal Federalism: The Implications of Fiscal Incentives." Journal of Urban Economics 65 (3): 279–293.

World Bank. 2019. Pakistan@100: Shaping the Future. Washington, DC: World Bank.

Zaidi, S. Akbar. 2015. Issues in Pakistan's Economy: A Political Economy Perspective. 3rd ed. Karachi: Oxford University Press.

Zhang, G. Peter. 2003. "Time Series Forecasting Using a Hybrid ARIMA and Neural Network Model." Neurocomputing 50: 159–175.

Appendix A. Model hyperparameters

Table A1. Hyperparameters and Training Protocol of the Learning Models.

Model	Architecture / settings	Regularization and selection
Deep MLP ensemble	Feed-forward ReLU network; candidate architectures (32-16-8) and (16-8); Adam, learning rate 0.003, up to 1,500 epochs; 10-seed ensemble	L2 penalty $\alpha \in \{1, 10, 50\}$ chosen on the final year of each training window; inputs and target standardized on training data only
Hybrid Ridge–DNN	Ridge ($\alpha = 3$) linear base; DNN fitted to ridge residuals with the same search grid	As above
LSTM	One recurrent layer, 6 hidden units, window of 2 years, 5 input channels; implemented in NumPy with back-propagation through time	L2 = 0.005; Adam (0.005), 600 epochs; 8-seed ensemble; forget-gate bias initialized at 1

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Model	Architecture / settings	Regularization and selection
Softmax inverse-formula layer	4 logits → softmax weights → share = w′M	Adam (0.05), 6,000 iterations; 300 year-block bootstrap fits
Random forest / gradient boosting	500 trees, min. leaf 3 / 200 trees, depth 2, learning rate 0.05, subsample 0.8	Fixed a priori
DML nuisance DNN	(32-16) ReLU, α = 1, 5-seed average	Four year-block cross-fitting folds
Shapley attribution	Interventional permutation sampling, 60 permutations per observation, 16 background rows	Applied to the full-sample 10-seed DNN ensemble

Notes: All random seeds are fixed; the complete code reproduces every table and figure from the dataset workbook.

Appendix B. Province-Year Panel

Table B1. Province-Year Panel used in the Analysis.

Fiscal year	Type	Total (Rs bn)	Punjab	Sindh	KP	Balochistan	pc Punjab / Bal. (Rs '000)
1997-98	A	100	57.84 (-0.04)	23.28 (-0.00)	13.59 (+0.05)	5.29 (-0.01)	0.8 / 0.8
1998-99	A	106	57.89 (+0.01)	23.31 (+0.03)	13.53 (-0.01)	5.26 (-0.04)	0.8 / 0.9
1999-00	A	121	57.89 (+0.01)	23.29 (+0.01)	13.54 (+0.00)	5.28 (-0.02)	0.9 / 0.9
2000-01	A	136	57.90 (+0.02)	23.29 (+0.01)	13.52 (-0.02)	5.29 (-0.01)	1.0 / 1.0
2001-02	A	140	57.84 (-0.04)	23.34 (+0.06)	13.53 (-0.01)	5.30 (-0.00)	1.0 / 1.0
2009-10	B	615	53.01 (-4.35)	24.94 (+1.23)	14.88 (+1.06)	7.17 (+2.06)	3.5 / 4.7
2011-12	BE	1,044	50.85 (-0.89)	24.13 (-0.42)	16.09 (+1.47)	8.93 (-0.16)	5.5 / 9.2
2013-14	A	1,406	45.95 (-5.79)	27.31 (+2.76)	16.64 (+2.02)	10.10 (+1.01)	6.4 / 13.1
2014-15	A	1,539	47.24 (-4.50)	26.38 (+1.83)	16.31 (+1.69)	10.07 (+0.98)	7.0 / 13.9
2015-16	A	1,861	48.41 (-3.33)	26.22 (+1.67)	16.23 (+1.61)	9.13 (+0.04)	8.5 / 14.7
2016-17	A	1,966	47.25 (-4.49)	26.30 (+1.75)	16.02 (+1.40)	10.43 (+1.34)	8.6 / 17.2

JOURNAL FOR SOCIAL SCIENCE STUDIES
An International Peer-Reviewed Open Access Journal

ISSN (Online): 3006-2071 | ISSN (Print): 3006-2063
Volume 4 • Issue 7 • November 2026

Fiscal year	Type	Total (Rs bn)	Punjab	Sindh	KP	Balochistan	pc Punjab / Bal. (Rs '000)
2017-18	A	2,217	48.67 (-3.07)	25.35 (+0.80)	16.37 (+1.75)	9.61 (+0.52)	9.8 / 17.3
2018-19	A	2,398	48.67 (-3.07)	25.02 (+0.47)	16.39 (+1.77)	9.92 (+0.83)	10.3 / 18.7
2019-20	A	2,503	47.82 (-3.92)	24.53 (-0.02)	16.02 (+1.40)	11.63 (+2.54)	10.4 / 22.2
2020-21	A	2,741	49.03 (-2.71)	25.10 (+0.55)	16.27 (+1.65)	9.60 (+0.51)	11.3 / 19.4
2021-22	A	3,510	49.57 (-2.17)	24.87 (+0.32)	16.38 (+1.76)	9.17 (+0.08)	14.3 / 23.0
2022-23	BE	4,099	49.50 (-2.24)	25.11 (+0.56)	16.36 (+1.74)	9.03 (-0.06)	16.3 / 25.7
2023-24	BE	5,454	49.85 (-1.89)	24.67 (+0.12)	16.41 (+1.79)	9.07 (-0.02)	21.3 / 33.2
2024-25	BE	7,438	49.68 (-2.06)	24.92 (+0.37)	16.42 (+1.80)	8.98 (-0.11)	28.2 / 43.5
2025-26	BE	8,206	49.67 (-2.07)	24.91 (+0.36)	16.36 (+1.74)	9.06 (-0.03)	30.4 / 46.9
2026-27	BE	8,847	49.76 (-1.98)	24.95 (+0.40)	16.31 (+1.69)	8.99 (-0.10)	32.0 / 48.6

Notes: Shares in % with deviation from formula share (pp) in parentheses. A = actual; BE = budget estimate; B = 7th NFC benchmark (FY2009-10). FY2023-24 onward budget figures include straight transfers.